Chapter 4

A Game Theory Approach to the Effect of Piracy on Collusion in a Vertically Differentiated Oligopoly

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4.1. INTRODUCTION[1]

In Italy, seizures of large amounts of fakes and falsified items occur every day: in 2013 alone, Guardia di Finanza, the Italian law enforcement agency under the authority of the Minister of Economy and Finance, seized 22 million fakes that, according to some technicians, were indistinguishable from the original[2].

These fakes were ready to increase an illegal black market that, in 2010 alone, was valued 18 billion Euro[3]. Usually fakes and counterfeit goods are sold along some streets and in local marketplaces and bazaars of some Italian cities, mainly in the South.

One of the reasons for such a presence of falsified items in the South of Italy, and more specifically in the Neapolitan area, can be seen in the specificity of the productive chain of the fashion sector: big fashion brands give raw material to some specialist craftsman in this area, which pirates part of the production, to sell it on the fake market[4].

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[1] The author wishes to acknowledge Katarzyna Metelska-Szaniawska, an anonymous referee and the participants at the 2013 Annual Meeting of the European Association of Law and Economics (EALE) held in Warsaw, where a previous and more technical version of this paper was presented.

[2] Source Guardia di Finanza website: http://www.gdf.gov.it/GdF/it/Stampa/Comunicati_stampa/Comunicati_stampa_del_2014/Gennaio_2014/info-950081271.html

[3] Source Confindustria, the Italian employers federation.

[4] An interesting and accurate description of way the big fashion brands outsource the production is in the best seller book Gomorrah, by Roberto Saviano: "The auctions the big Italian

In recent years, economic literature has deeply analyzed piracy and copyright violation. Nevertheless most of the contributions focus on the study of digital markets and monopoly. In this paper I use a Game Theory approach to describe the effect the entry of a pirate may have in a vertically differentiated duopoly where, originally, two firms compete producing a high quality and a low quality good.

The case study I refer to is the fashion market, which has a certain centrality in the debate between law scholars, while a paper in the law literature that uses a similar approach is Wong (2012). Grassi (2014) presents the same arguments in a more technical way.

The issue whether fashion design merits extended legal protection has generated much debate among law scholars.

Raustiala and Sprigman (2006) assert that additional legal protection is unnecessary for the fashion market and that copying is beneficial to the fashion industry because in some way it speeds the fashion cycle.

A number of scholars have rejected Raustiala and Sprigman argument: Myers (2009) notes that they underestimate the new technologies of copying; Howard (2009) contests the sociology of the modern fashion customer used by Raustiala and Sprigman, which misunderstands the motivations of consumers using outdated assumptions about the fashion industry. Hendrick (2008) notes that fashion design protection would likely create more trouble than value because fashion design is hard to define and equally difficult to protect. Moreover if a designer were able to legally protect an article of apparel, then design pirates could easily avoid infringement by slight variations to the original design, leaving the original designer holding virtually worthless rights for the protected design[5]. Hemphill and Suk (2009) propose to limit protection to the very close imitations, as an intermediate stand between permitting free copying of fashion design and creating a broad right of exclusion. Harchuck (2010) underlines that it is a matter of fact that the fashion industry is losing billions of dollars every year because of piracy, and hence something needs to be done to protect these creations: fashion has become an entity, law needs to recognize this, as it has done for so many other art forms.

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brands hold in this area are strange. No one wins the contract and no one loses. The game consists in entering or not entering the race. Someone throws out an offer, stating his time and price. If his conditions are accepted, he won't be the only winner, however. His offer is like a head start the others can try to follow. When the brokers accept a bid, the other contractors decide if they want in; whoever agrees gets the fabric. It's sent directly to the port of Naples, where the contractors pick it up. But only one of them will be paid: the one who delivers first, and with top quality merchandise. The other players are free to keep the fabric,but they don't get a cent. The fashion houses make so much money that material isn't a loss worth considering. Even the contractors who don't satisfy the requirements of the designer labels manage to find a buyer. They sell the garments to the clans to be put on the fake goods market" (Saviano 2006, pp. 45–46).

[5] Hendrick (2008) states: "Any benefit that could ultimately be derived from this sliver of protection is quickly negated by complications caused by trying to enforce exclusive rights against infringers. The cost of arguing whether a second design is substantially similar to the original design is significant in terms of time and money. Additionally, by the time a court reaches a final decision, the fashion design will likely no longer be in vogue" (p. 272).

On the contrary the analysis of the fashion market has been sidelined in the major debate in the economic literature even if piracy was a widely studied phenomenon because, since the last years of the twentieth century, the diffusion of Internet and broadband has facilitated new forms of technological piracy and copyright violation. A survey of this literature is provided by Peitz and Waelbroeck (2006).

Most analyses proposed in recent years in the economic literature have two common assumptions:

- they focus on the piracy of digital products
- they concentrate on the entry of a pirate into a monopolistic market

The growth of studies on digital piracy is undoubtedly connected to the diffusion of this kind of copyright violation; nevertheless, the absence of analysis on the fashion market, another major economic sector where diffusion of fakes and counterfeits is huge, is quite surprising[6].

One reason may be the second assumption of the recent economic models on piracy: in the typical scheme of the works on piracy a pirate enters a monopoly producing a low quality good (the copy), that can be more or less similar to the original, turning the market in an oligopoly with vertical differentiation[7].

The main goal of this study is to suggest a topic that has not been analyzed enough by economic theory and that, to the best of my knowledge, has been ignored by legal literature: the effect piracy may have on the ability of the firms to collude, in a vertically differentiated oligopoly.

I show that, in some cases, payoffs of firms might increase with piracy, since piracy may support collusion between the two firms producing the original goods and the collusive profits of the firms on a smaller market (the one with piracy) are bigger than the profits of Nash on a bigger market (the one without piracy)[8]. This result may explain the reason why in some markets, such as the fashion market, where the producers of the original brands basically control the supply chain of the sector, piracy and production of high quality fakes is huge, and this may confirm the intuition of some law scholar that fashion firms do not need a specific form of IP protection, since in some cases piracy might benefit the producer of the original good[9].

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[6] One exception is Harbi and Grolleau (2008).

[7] A relevant exception is Belleflamme and Picard (2007) who analyze piracy in an oligopolistic framework.

[8] Moreover the present article is part of the existing literature on the sustainability of collusion in vertical differentiated markets.

[9] The usual argument, advanced in opposition to the models of piracy that show how piracy can benefit some industry, is asking why the producers of the original good do not enter themselves in the low quality segment of the market selling a low quality legal substitute. In the fashion market this is not the case simply because we can expect that, given a low quality "for sure" good produced by the brand and a fake "maybe is an original" good produced by the pirate, consumers will always prefer the latter to the first. For a more technical discussion see note 24.

4.2. COLLUSION AS AN APPLICATION OF THE PRISONER'S DILEMMA

Collusion usually exists where agreements between incumbent firms exist. Thus firms can share what would otherwise be monopoly supernormal profits between them, acting as a monopoly. Nevertheless, it is not easy to implement tacit collusion in an oligopolistic market, since collusion represents a typical case of the Prisoner's Dilemma.

The prisoner's dilemma is the most basic Game Theory example. In the traditional version of the game, the police have arrested two suspects and are interrogating them in separate rooms. Each can either confess, thereby implicating the other, or keep silent. No matter what the other suspect does, each can improve his own position by confessing. If the other confesses, then one had better do the same to avoid harsh sentence that awaits a recalcitrant holdout. If the other keeps silent, then one can obtain favorable treatment accorded a state's witness by confessing. Thus, confession is the dominant strategy for each, but when both confess, the outcome is worse for both than when both keep silent.

Table 1 illustrates the typical Prisoner's Dilemma game matrix: 1 and 2 are the two players, C and NC their strategies, while the values in the matrix represent the sentences under the different strategies.

Table 1
The Prisoner's Dilemma Matrix

		2	
		C	*NC*
1	*C*	10; 10	0; 20
	NC	20; 0	1; 1

Source: Author's simulation.

The prisoner's dilemma has applications to economics, law, business, everyday life etc. One of the application is referred to as the case of tacit collusion between firms in oligopoly: as in the prisoner dilemma, each firm has the incentive to deviate from the collusive path and the final equilibrium is the Nash equilibrium, where each firm does not collude.

At least two structural aspects of the prisoner's dilemma deserve emphasizing: first, the game is simultaneous, that is the two players simultaneously decide their strategy, without observing the behavior of the other part, but having a "conjecture" on it; second, the game here described is one-shot, that is the two players meet each other just once. This cannot be the case of two firms which compete every day in the same market for years. These kinds of games are known as repeated games, and if there is uncertainty as to when the game ends, they are known as infinitely repeated games. In this case the equilibrium can change: the criminals may have the incentive to adhere to the strategy of not confessing, the firms to the one of colluding.

They may then be able to maintain higher prices by tacitly agreeing that any deviation from the collusive path would trigger some retaliation. In order to be sustainable retaliation must be sufficiently likely and costly to outweigh the short-term benefits from cheating on the collusive path: Friedman (1971) shows that many schemes can make collusion possible.

A simple form of retaliation consists in the breakdown of collusion and the restoration of normal competition profits: firms trust each other to maintain collusive prices, but if one of them deviates, trust vanishes and all firms start acting in their short run interest. In such a case collusion is sustainable as equilibrium if firms are sufficiently patient, that is if the critical value of the discount factor of the future profits is large enough.The intuition is that firms must put sufficient weight on future losses to offset the temptation of deviating: the cheater's reward comes at once, while the loss from punishment lies in the future. If players heavily discount future payoffs, then the loss may be insufficient to deter cheating. Thus, cooperation is harder to sustain among very impatient players (one classical example is found in Political Science where a player who underestimates future payoffs is the government).

Formally, let π^N, π^C, π^D be respectively the per period payoff in the Nash equilibrium of the one shot game, in collusion and deviating from collusion undercutting the rival. Then, for collusion to be sustainable it must be:

$$\frac{\pi^C}{1-\delta^*} \geq \pi^D + \frac{\delta^* \pi^N}{1-\delta^*}$$

That is

$$\delta^* \geq +\frac{\pi^D - \pi^C}{\pi^D - \pi^N} \equiv \sigma_i \tag{1}$$

where:
δ^* is the discount factor, common for all firms, and
σ_i is the critical discount factor for firm *i*.

Thus, the higher the value of σ_i, the more difficult the sustainability of collusion is[10].

In the presence of a differentiated product, all profits are affected by the degree of product differentiation and it is possible to analyze the relationship between product differentiation and cartel sustainability.

As originally showed by Deneckere (1983), differentiation between brands affects the scope of collusion in two ways: firstly, it limits the short term gains from undercutting rivals, since it becomes more difficult to attract their

[10] For example it is easy to show that, in a duopoly with linear demand, collusion is easier under price competition ($\sigma_i = 0.5$) than under quantity competition ($\sigma_i \cong 0.53$).

customers; secondly, it also limits the severity of price wars and thus the firms ability to punish a potential deviation.

In other words, in differentiated markets, sustainability of collusion depends on two effects: the first tends to support this behavior since as differentiation decreases competition between brands increases, the second tends to encourage deviation since as differentiation decreases, gain from deviation increases.

Most of the authors involved in debate on sustainability of collusion in differentiated market (see, *inter alia*, Deneckere (1983), Chang (1991), Ross (1992) and Hackner (1994); more recently Lambertini (2000) and Andaluz (2010)) show that under price competition collusion is not stable, since, as products become more substitutable, a deviation from collusion becomes increasingly attractive.

Therefore, in markets where there is close substitutability between the original goods, collusion is harder. Nevertheless the presence of a pirate may undermine this condition, making collusion easier.

4.3. THE MODEL WITHOUT PIRACY

Economic theory distinguishes two kinds of differentiation: horizontal and vertical.

Broadly speaking, we have horizontal differentiation when goods are different but at the same price some consumers will buy one and some will buy other, it really depends on their preferences: for example Pepsi and Coca Cola, or Gucci and Versace top of the line products.

The classical economic model used to describe this case is the Hotelling (1929) model and the linear beach: in each extreme of the beach, there is an ice-cream post. Although the two posts sell exactly the same ice creams, consumers are not indifferent between them, they prefer to buy from the one that is closer. According to the hypothesis on the transportation costs we can have maximum rather than minimum differentiation between the goods (i.e. in the localization of the ice cream shop).

On the contrary,vertical differentiation occurs in a market where several goods can be ordered according to their objective quality from the highest to the lowest. In other words two goods A and B are vertically differentiated when, if the two goods have the same price, all the consumers prefer good A to good B: in this case it is possible to say that good A is better than good B: think, for example, of a Gucci bag and its equivalent sold in a store such as Zara.

One model used to describe vertically differentiated markets is the one introduced in the literature by Mussa and Rosen (1978): assume that two products of different quality, say 1 and 2, are available for consumption. Consumer has the following utility function:

$$U = \begin{cases} \theta - p & \text{if they purchase the high quality goods} \\ s\theta - q & \text{if they purchase the low quality good} \\ 0 & \text{if they do not purchase any good} \end{cases}$$

where:
p and q are the prices of the goods;
θ is a parameter describing the intensity of the preferences of the consumer through the quality, it is uniformly distributed over $[\bar{\theta}\,;\,\underline{\theta}]$, with density 1; the quality of the high quality good is normalized to 1 and s ≤ 1 set the difference in quality between the two good: $s = 1$ means that the two goods have the same quality, hence we concentrate on the case where $s < 1$.

Being $\theta_i = \dfrac{p-q}{1-s}$ the consumer who is indifferent between the high quality and the low quality good, and $\theta_j = \dfrac{q}{s}$ the consumer who is indifferent between purchasing the low quality good and nothing, the demands for the high quality good (D_1) and low quality good (D_2) are the standard linear demands[11]:

$$D_1 = \bar{\theta} - \frac{p-q}{1-s} \qquad D_2 = \frac{p-q}{1-s} - \frac{q}{s} \tag{2}$$

The fashion market is clearly an oligopoly where both horizontal and vertical differentiation exist: as in the horizontal differentiated markets, fashion brands often have an amazingly rich combination of shapes, colors, materials etc. Here we concentrate on the vertical differentiation of this sector.

Moreover, in order to simplify the model, I assume that firms maximizes their revenues, i.e. I normalize to zero their cost function: this seems to be a consistent assumption with all the industry where the main cost is the fixed cost of developing new products or, as in the case of fashion market, new lines of clothing, footwear, accessories etc.

For computational purpose I assume $\theta = 100$ and an intermediate valor of s, $s = .5$, which represent an intermediate substitutability between the two goods[12].

In such a case the demand functions are given by

$$D_1 = 100 - 2p - 2q \quad \text{and} \quad D_2 = 2p - 4q$$

It is hence easy to obtain the Nash equilibrium in this game, which is given by

[11] Setting $A = \dfrac{1}{1-s}$ and $B = \dfrac{2s-1}{s(1-s)}$ these equation can be expressed as $D_1 = \theta - Ap + Aq$ and $D_2 = Ap - Bq$.

[12] A generalization of this model is in Appendix.

$$\pi_1^N = 1632.7; \quad \pi_2^N = 204; \quad p = 28.5; \quad q = 7.14$$
$$D_1 = 57.14 \quad \text{and} \quad D_2 = 28.57$$

In oligopoly each firm knows that its profits depend not only on its own actions, but also on the actions of its rivals. It is possible that firms, each possessing this insight and understanding that its competitors all possess it, might be able to succeed in the implementation or even the establishment of a conduct which maximizes the common profits: it is the case of tacit collusion, where the monopolistic outcome can arise spontaneously.

In a standard Cournot oligopoly, in order to implement tacit collusion, it may be sufficient that one firm reduces the production with respect to the Nash equilibrium: in a repeated game such a reduction can be interpreted by the other firms as a signal of willingness to collude. In a vertically differentiated oligopoly tacit collusion is more difficult to implement, since there is quality differentiation between the goods and hence it is difficult to send this kind of signal. Moreover, it is easy to show that a multiproduct monopolist, i.e. a monopoly which can produce goods of different quality, maximizes its profits producing the high quality goods only[13].

Hence, collusion in a vertically differentiated oligopoly takes the form of the exit of the firms producing the low quality goods from the market, exit which has to be rewarded, and hence it is difficult to such a reward to be tacit.

So, following the economic literature on collusion in differentiated oligopoly[14], we assume that under collusion each firm obtains profits equal to the profits derived from the non-cooperative equilibrium, and the remaining surplus from cooperation is split equally between the firms: this is known as the Nash Bargained Solution to the cooperation problem between the firms[15].

We have that $\pi_i^C = \frac{1}{2}\left(\pi_{i+j}^M + \pi_i^N - \pi_j^N\right)$, where π_{i+j}^M is the profit of the multiproduct monopolist and i and j are the two firms.

Given the demand functions expressed in the numerical simulation it is easy to show that

$$\pi_{i+j}^M = 2500, \quad p = 50, \quad D_1 = 50 \quad \text{and, obviously,} \quad D_2 = 0.$$

[13] Consider the demand functions expressed in equation (2). The profit of a multiproduct monopolist would be given by: $\pi_M = \left(\bar{\theta} - \frac{p-q}{1-s}\right)p + \left(\frac{p-q}{1-s} - \frac{q}{s}\right)q$. Maximizing with respect to p and q, we have $p = \frac{1}{2}$ and $q = \frac{1}{2}s$, that substituted in the demand function of the low quality good lead us to $D_2 = 0$. For a more formal treatment see also Lambertini et al (2006).

[14] See for example Andaluz (2010).

[15] A slightly different solution to this cooperation problem is the Shapley Value. Both the Nash Bargaining Solution and the Shapley Value are widely applicable concepts for solving these games. For an exhaustive treatment of bargaining problem see Osborne and Rubinstein (1994).

Hence the profits of the two firms in case of collusion are

$$\pi_1^C = 1964.4; \quad \pi_2^C = 535.65.$$

In the standard oligopoly with homogenous products, deviating from collusion means producing more than the agreement (in case of Cournot competition) or undercutting the rival (in case of Bertrand competition).

In a vertically differentiated oligopoly, for firm 1 it consists in keeping all the monopolistic profit, for firm 2 it means reentering the market selling a low quality good at a price that is set as the best response to the monopolistic price set by firm 1.

Hence we have that, in case firm 1 deviates $\pi_1^{D_1} = 2500$ and $\pi_2^{D_1} = 0$.

In case it is firm 2 to deviate $\pi_1^{D_2} = 1250$ and $\pi_2^{D_2} = 625$, in addition to $p = 50$, $q = 12.5$, $D_1 = 25$ and $D_2 = 50$

Table 2 represents this "collusion game" in the typical Game Theory matrix:

Table 2
The Collusion Game, when $s = .5$

		2	
		D	*C*
1	*D*	1632.7; 204	2500; 0
	C	1250; 625	1964.4; 535.65

Source: Author's simulation.

D and *C* represent the two possible strategies for the firms: Deviating and Cooperating. As in a typical prisoner's dilemma, in the one shot game both firms have a dominant strategy that is deviating from the collusive agreement, hence there is a Nash equilibrium where both firms deviate, which is Pareto inferior with respect to the case where both firms collude.

Nevertheless in an infinitely repeated framework this equilibrium can change, this happens according to the value of the discount factor σ expressed in equation (1): the lower this factor is, the easier the collusion.

Since we have two asymmetric firms, we have two values of the discount factor:

$$\sigma_1 = \frac{\pi^D - \pi^C}{\pi^D - \pi^N} = \frac{2500 - 1964.4}{2500 - 1632.7} = 0.62$$

$$\sigma_2 = \frac{\pi^D - \pi^C}{\pi^D - \pi^N} = \frac{625 - 535.65}{625 - 204} = 0.21$$

The relevant discount factor, i.e. the one that ensures the sustainability of the collusive equilibrium, is given by the maximum between the ones of the two firms, hence in this case by the one of firm 1, $\sigma_1 = 0.62$[16].

4.4. HOW PIRACY MIGHT AFFECT COLLUSION

I am now introducing piracy into the model. The pirate makes low quality imitations of the original products, and sells it illegally on the market at a low price, thereby competing with the two firms which produce the original goods, and in particular with firm 2, which demand is directly affected by the pirate.

The entry of a pirate modifies the utility function of the consumer. The utility to consume the fake can be described by $U_F = v\theta - w$, where v is the quality discount factor of the fake and wits price. Many market configurations are possible: for example we could have $v = 1 > s$. In such a case the quality of the fake is equal to the one of the high quality good, and superior to the one of the second good. Since the price of the fake is lower than the price of the original goods, in this case we would have the annihilation of the market. In some way this case may describe the evolution of the music market in the last decade, after the diffusion of file-sharing programs[17], and better fits with the digital markets, where the pirate good is very close in quality to the originals: with digital technology, most modern piracy involves an exact and perfect copy of the original made from a hard copy or downloaded over the Internet.

In this framework I concentrate on the case where $v \leq s \leq 1$, i.e. the case where the quality of the fake is inferior to both original goods, and hence both firms can be affected by the pirate, but continue to produce.

As in the previous paragraph we consider the marginal consumers, which are three: $\theta_i = \frac{p-q}{1-s}$ the consumer who is indifferent between the high quality and the low quality good; $\theta_j = \frac{q-w}{s-v}$ the consumer who is indifferent between the low quality good and the fake; $\theta_x = \frac{w}{v}$ the consumer indifferent between the fake and nothing. Hence we have three demand functions[18]:

[16] For $s = .5$, which describes an intermediate level of substitutability between the two goods, firm 2 has more to earn from a collusive agreement than firm 1. Things change for high level of substitutability. It is possible to show that for $s > .8$ to be relevant for sustainability is the value of the discount factor of firm 2.

[17] Such a market still exists, but in the last decade it has completely changed its business and its dimension. According to the International Federation of the Phonographic Industry (IFPI) worldwide revenues for CDs, vinyl, cassettes and digital downloads fell from \$36.9 billion in 2000 to \$15.9 billion in 2010. For more details see the annual reports issued by IFPI at http://www.ifpi.org/digital-music-report.php.

[18] In this vertical model the demand of the high quality good producer is not directly affected by the pirate.

$$D_1 = \bar{\theta} = \frac{p-q}{1-s} \qquad D_2 = \frac{p-q}{1-s} - \frac{q-w}{s-v} \qquad D_p = \frac{q-w}{s-v} - \frac{w}{v} \tag{3}$$

where:
D_1 and D_2 are the demands of the producers of the original goods, while D_P is the demand of the pirate firm.

For computational purpose I continue to assume $\theta = 100$, $s = 0.5$, and I consider $v = 0.2$, i.e. the quality of the fake is sensibly lower than the quality of the second original good.

In this case the demand functions are:

$$D_1 = 100 - 2p - 2q, \qquad D_2 = 2p + \frac{10}{3}w - \frac{16}{3}q, \qquad D_p = \frac{10}{3}q - \frac{25}{3}w$$

It is just a matter of computation to obtain the Nash equilibrium in this game, which is given by

$$\pi_1^N = 1543; \qquad \pi_2^N = 164.61; \qquad p = 27.78; \qquad q = 5.5$$
$$D_1 = 55.55 \qquad D_2 = 29.63$$

I consider now the case that the two firms producing the original good collude. The collusive profit is described by:

$$\pi_{1+2} = \left(\bar{\theta} - \frac{p-q}{1-s}\right)p + \left(\frac{p-q}{1-s} - \frac{q-w}{s-v}\right)q$$

Given $s = 0.5$ and $v = 0.2$,and taking into account the behavior of the pirate, which maximizes its profits, it is easy to show that

$$\pi_{1+2} = 2175$$

Applying the same sharing rule used in the previous section, it turns out that the collusive profits of firm 1 and 2 are given by

$$\pi_1^C = 1776.8; \qquad \pi_2^C = 398.2$$

Consider now the case the two firms decide to deviate from collusion. If this happens, each firm sets its price as best reply to the collusive price set by the other firm, given the competitive behavior of the pirate.

The best reply function of firm 1 is described by $p = \frac{100 - 2q}{4}$. If firm 2 keeps the collusive price $q = 16.67$, we have that $\pi_1^{D_1} = 2083$ and $\pi_2^{D_1} = 92.5$.

Analogously the best reply function of firm 2 is described by $q = \frac{3p + 5w}{16}$. If firm 1 keeps the collusive price $p = 41.67$ we have that $\pi_1^{D_2} = 1388.8$ and $\pi_2^{D_2} = 370.43$.

Table 3 represents this modified collusion game with piracy in the typical Game Theory matrix:

Table 3
The Collusion Game in case of Piracy, when *s* = .5, *v* = .2

		2	
		D	*C*
1	*D*	1543; 154.61	2083; 92.5
	C	1388.8; 370.43	1776.8; 398.2

Source: Author's simulation.

First of all note that the presence of a pirate has sensibly modified the strategies of firm 2: now it does not have a dominant strategy, since "Collude" permits it to obtain bigger profit with respect to "Deviate", in case firm 1 keeps the collusive agreement[19]. On the contrary Deviate remains the dominant strategy for firm 1, and hence the Nash equilibrium is the Pareto inferior one where both firms deviate.

Let me turn to the discount factor, that is evidently the one of firm 1, since for firm 2 there is no defection prize. In this case we have

$$\sigma_1^P = \frac{\pi^D - \pi^C}{\pi^D - \pi^N} = \frac{2083 - 1776.8}{2083 - 1543} = 0.56$$

With this values, the discount factor in case of piracy is smaller than the one in a market without piracy, hence, according to the economic theory, collusion is easier.

It is possible to show that this result is quite general, and it is valid for most of the value of *s* and *v*[20], especially that it is always true when there is a close substitutability between the original goods and the fake is sensibly worse than the original: in terms of a vertically differentiated model when *s* is close to 1 and *v* is small enough.

Table 4 compares the values of the critical discount factor without piracy (first column) with the critical discount factor in presence of piracy for some values of *s* and *v*.

In a vertically differentiated oligopoly the presence of a low quality cheaper substitute on the market, attracts consumers with lower reservation prices and, to a certain extent, enlarges the market for the product. When this substitute good is a fake, i.e. when the entrant is a pirate, this may cause damages to the producer of the original goods: this is probably true in markets for digital good, where the copy is substantially equal to the original. For example, the diffusion of MP3 files has totally changed the music industry after

[19] This is due to the asymmetry in the profits of the two firms.
[20] See Grassi (2014).

the creation of file-sharing software like Napster[21] and the recent diffusion of software like Popcorn Time, that allows to watch in streaming movies and TV series, is expected to modify this business.

Table 4
Comparison between the values of the critical discount factors in a no piracy framework (column 1) and in a world with piracy, for given values of *v* and *s* (remaining columns)

	σ^{NP}	$\sigma^{P}_{V=.2}$	$\sigma^{P}_{V=.5}$	$\sigma^{P}_{V=.7}$
$s = .3$	.67	.59		
$s = .5$	.62	.56		
$s = .8$	.551	.541	.549	.55

Source: Author's simulation.

On the contrary the effect of a fake in a non-digital market may be different. Consider for example the fashion market: it is apparently a market where a close substitutability between the original goods exists[22], and hence, according to the economic theory, collusion between the firms should be harder. Nevertheless, in this market in case of piracy the fake good is judged worse than the original: a Gucci and a Versace five hundred Euro bags are both considered very different from a twenty-five Euro unbranded bag.[23] In terms of our model the value of *s* is higher than the value of *v*.

In this case the difference in quality between the original and the copy leaves to the seller of the original goods the consumers more willing to spend money, i.e. with an inelastic demand, and this may enforce the willingness to collude.

In other words, the drop in demand of original goods, even if slight, may cause a reduction for the temptation to deviate from the collusive path, and this reduction is particularly strong when the quality of the pirate good is low.

This is described in detail in Table 4 that shows the effects of piracy on collusion: the first column represents the value of the discount factor in a world

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[21] Napster was the first software allowing people to share on the net their music folder. It appeared in the right moment: the digitalization of music was complete, after the CDs had completely replaced LPs and tapes, most of the families in the developed countries had a computer, and most of these computers were connected to the Inernet. Although record industries immediately understood the danger coming from Napster, it took two years to block the service: Napster went live on June 1, 1999; on December 7, 1999 RIAA sued Napster for copyright infringement; on July 11th 2001 Napster was definitively interrupted. Within two years file-sharing became a massive activity and new and more efficient software was programmed that completely changed the music indutry.

[22] We can infer this comparing the prices of top of the line products of different brands in the fashion market. For example, latest Gucci handbags are priced similarly to comparable Prada products: both are sold for over 2100 Euro. According to economic theory, this implies high cross elasticity and hence close substitutability between the two goods. The same argument holds true for the entire product line.

[23] Following the argument expressed in footnote 16.